Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$=\frac{n}{2}(n-3)+\frac{n(n-3)\,(n^2-3n-2)}{8}=\frac{n(n-1)\,(n-2)\,(n-3)}{8}.$$

II. Solution by the PROPOSER.

A *complete* polygon of n sides is formed by n intersecting lines, and the first question is, how many lines will join the points of intersection of n intersecting lines. n lines can intersect in $\frac{n(n-1)}{2}$ points. From each of these points there can be drawn the other $\frac{n(n-1)}{2}-1$ lines. Therefore, from $\frac{n(n-1)}{2}$ points, $\frac{n(n-1)}{2}\left[\frac{n(n-1)}{2}-1\right]$, of which, evidently, only half can be taken; consequently, the required number of lines is $\frac{1}{8}(n+1)n(n-1)\,(n-2)$. Since each line drawn from $\frac{n(n-1)}{2}$ points cuts $n-2$ lines, we must subtract $\frac{n(n-1)}{2}(n-2)$ from the last result, in order to obtain the number of diagonals of a complete polygon of n sides. Therefore the required number of diagonals is

$$\tfrac{1}{8}(n+1)n(n-1)\,(n-2)-\tfrac{1}{2}n(n-1)\,(n-2)$$
$$=\tfrac{1}{8}n(n-1)\,(n-2)\,(n-3),$$

or, more briefly, $3\binom{n}{4}$.

Also solved by Jeannette Brooks and A. H. Holmes.

CALCULUS.

302. Proposed by PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.

Find an algebraic integral of
$$LCy''+[18L(s-x^2)-2kC]y'+[L(\tfrac{2}{3}k-10x)-8k(s-x^2)]y=0,$$
where $L=2kx+3s-k^2$, $C=12sx-4x^3-t$, $k^4-6sk^2-tk-3s^2=0$. The roots of the determining equation are 0, $-\frac{1}{2}$ for a root of $C=0$; 0, 2 for a root of $L=0$; and 1 and $\frac{3}{2}$ at infinity.

No solution of this problem has been received.

303. Proposed by C. N. SCHMALL, New York City.

If $A=\int_0^1\frac{dx}{\sqrt{(1-x^4)}}$, express A in terms of Gamma-functions. [Part of ex. 39, p. 474, Bromwich's *Theory of Infinite Series.*]

I. Solution by JOSEPH A. NYBERG, Student, University of Chicago.

In the integral

$$u=\int_0^1 x^{\lambda-1}(1-x^{\mu})^{\nu-1}dx \tag{1}$$

make the substitution, $x^{\mu}=y$. Then

$$dx=\frac{1}{\mu}y^{(1/\mu)-1}dy,\quad \begin{cases} x=0,\ \dots,\ 1 \\ y=0,\ \dots,\ 1 \end{cases}$$

Therefore,

$$u=\frac{1}{\mu}\int_0^1 y^{(\lambda/\mu)-1}(1-y)^{\nu-1}dy\equiv\frac{1}{\mu}B\left(\frac{\lambda}{\mu},\nu\right) \tag{2}$$

by the definition of the beta function. Using the formula,

$$B(a,b)=\frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} \tag{3}$$

we get

$$u=\frac{1}{\mu}\,\frac{\Gamma(\lambda/\mu)\,\Gamma(\nu)}{\Gamma[(\lambda/\mu)+\nu]}. \tag{4}$$

(4) is the solution of (1) in terms of gamma functions.

On putting $\nu=\frac{1}{2}$, $\lambda=1$, $\mu=4$, we have

$$\int_0^1\frac{dx}{\sqrt{(1-x^4)}}=\frac{1}{4}\,\frac{\Gamma(\frac{1}{4})\,\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})}$$

Proofs of the formula,

$$B(a,b)=\frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$$

are found in Byerly's *Integral Calculus*, p. 114; in an article by A. Pringsheim in *Mathematische Annalen*, Band 31 (1888), p. 480; and in N. Nielsen's *Hankbuch der Theorie der Gamma Funktion*, p. 148.

II. Solution by FRANCIS RUST, C. E., Pittsburg, Pa.

Let $x^2=\sin\phi$. Then $dx=\dfrac{\cos\phi\,d\phi}{2\sqrt{(\sin\phi)}}$. Hence,

$$A=\tfrac{1}{2}\int_0^{\frac{1}{2}\pi}\sin^{-\frac{1}{2}}\phi\cos\phi\,d\phi=\tfrac{1}{4}B(\tfrac{1}{4},\ \tfrac{1}{2})=\tfrac{1}{4}\frac{\Gamma(\frac{1}{4})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})}=\tfrac{1}{4}\sqrt{(\pi)}.\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}.$$

In elliptic integrals, using Legendre's notation, $A=\frac{1}{2}\sqrt{(2)}F^I\,(\frac{1}{2}\sqrt{2})=1.311029$.

Also solved similarly by James A. Whitted, and the Proposer.

304. Proposed by **H. C. FEEMSTER, York College, York, Neb.**

Reduce $axyp^2+(x^2-ay^2-b)p-xy=0$ to Clairaut's form, and hence solve the equation.

Solution by **OLLINE CRUICKSHANKS, Bowling Green, Ky., and S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, New York.**

Expanding and factoring,

$$apy(px-y)+x(px-y)=bp,\quad (apy+x)(px-y)=bp. \tag{I}$$

Let $x^2=u$ and $y^2=v$. $\therefore 2xdx=du$ and $2ydy=dv$.

$$p=\frac{dy}{dx}=\frac{x}{y}\,\frac{dv}{du}.$$

Substituting this value of p in (I),

$$\left(\frac{x^2}{y}\,\frac{dv}{du}-y\right)\left(\frac{axy}{y}\,\frac{dv}{du}+x\right)=\frac{bx}{y}\,\frac{dv}{du}. \tag{II}$$

Multiplying (II) by y/x,

$$\left(x^2\,\frac{dv}{du}-y^2\right)\left(a\,\frac{dv}{du}+1\right)=b\frac{dv}{du}. \tag{III}$$

Since $x^2=u$ and $y^2=v$, (III) becomes

$$(u\frac{dv}{du}-v)(a\frac{dv}{du}+1)=b\frac{dv}{du}. \tag{IV}$$

$$\therefore v=u\frac{dv}{du}-\frac{b\frac{dv}{du}}{a\frac{dv}{du}+1}. \tag{V}$$

$\therefore v=uc-\dfrac{bc}{ac+1}$; but $v=y^2$ and $u=x^2$. $\therefore y^2=cx^2-\dfrac{bc}{ac+1}$.

Also solved by A. H. Holmes, who substitutes for p, xz/y, and then differentiates the resulting equation.